

M49A Notes for Transformations Section 1.7
Solutions for Examples in the Notes

To find graph of	Describe the transformation of the graph of $y = f(x)$
$f(x) - c, c > 0$	Vertical Translation: Shift down c units
$f(x) + c, c > 0$	Vertical Translation: Shift up c units
$f(x - c), c > 0$	Horizontal Translation: Shift right c units
$f(x + c), c > 0$	Horizontal Translation: Shift left c units
$cf(x), 0 < c < 1$	Vertical Shrink scale by factor of c
$cf(x), c > 1$	Vertical Stretch scale by factor of c
$f(cx), 0 < c < 1$	Horizontal Stretch scale by factor of $(1/c)$ <i>since $c < 1, 1/c > 1$</i>
$f(cx), c > 1$	Horizontal Shrink scale by factor of $(1/c)$ <i>since $c > 1, 1/c < 1$</i>
$-f(x)$	Reflect across y axis (all y values change sign)
$f(-x)$	Reflect across y axis (all x values change sign)

When doing more than one transformation, order matters.

Not following the correct order *may* give incorrect results

You can choose whether to do vertical followed by horizontal or horizontal followed by vertical.
But all the vertical transformations must be done in the order specified below.
And all the horizontal transformations must be done in the order specified below.

Vertical Transformations:

Reflections and stretches/compressions have first priority and can be done in any order.
 Do vertical shifts up or down after vertical reflections, stretches or compressions.

Horizontal Transformations:

Factor the expression inside the function.

$$\text{Examples: } (cx - d) = \left(c \left(x - \frac{d}{c} \right) \right) \quad (2x - 6) = (2(x - 3))$$

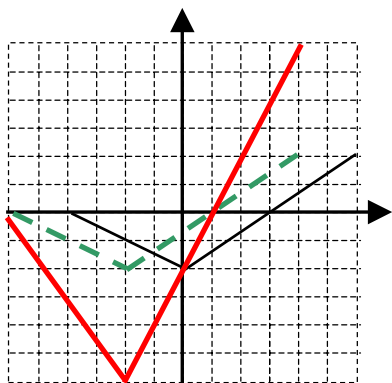
First shift function horizontally by d/c units (right if $d/c > 0$, left if $d/c < 0$)

Next do horizontal stretches/compressions using the line $x = d/c$ as the axis of reference

Then do horizontal reflections using the line $x = d/c$ as the axis of reference

For each problem the graph of $y = f(x)$ is shown. Describe the transformations of the graph of $y = f(x)$ to obtain the given function. Then use the transformations to draw the graph of the given function.

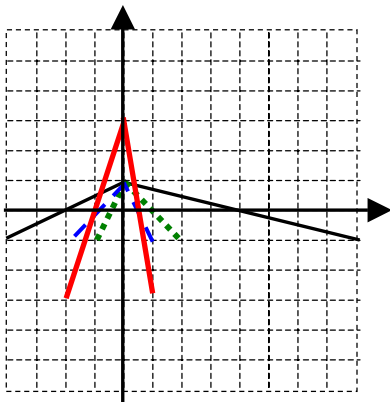
$$g(x) = 3f(x+2)$$



Shift 2 units to the left.

Vertical stretch by factor of 3

$$h(x) = 3f(-4x)$$

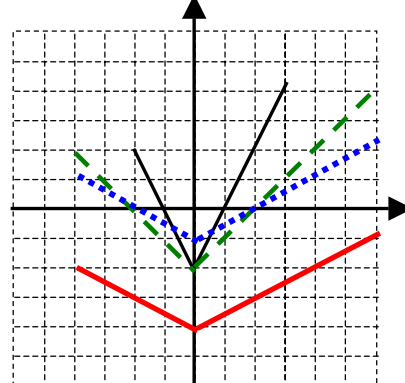


Horizontal Shrink by factor of 1/4

Reflect across y axis

Vertical Stretch by factor of 3

$$s(x) = .5f(.5x) - 3$$



Horizontal stretch by factor of 2

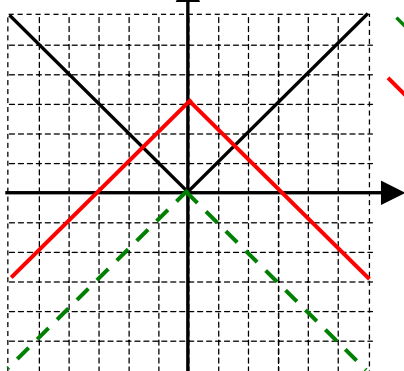
Vertical shrink by factor of 1/2

Shift down 3 units

Describe the transformations of the graph of $y = |x|$ to obtain the given function.

Then use the transformations to draw the graph of the given function.

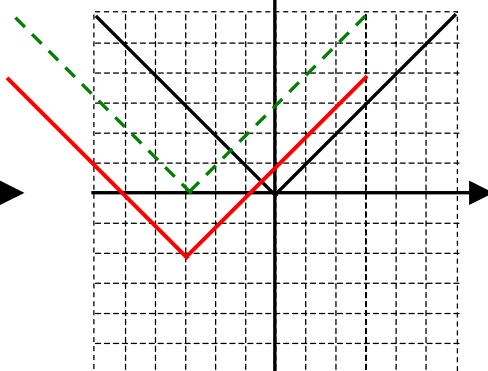
$$y = u(x) = -|x| + 3$$



Reflect across x axis

Shift upward 3 units

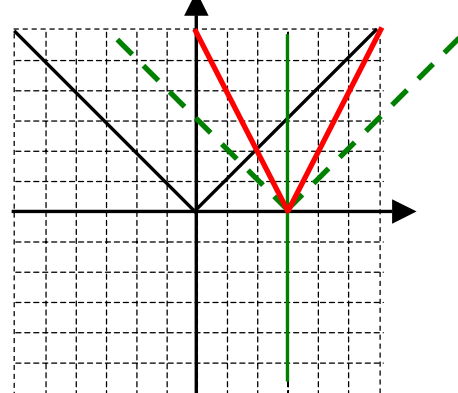
$$y = v(x) = |x+3| - 2$$



Shift left 3 units

Shift down 2 units

$$y = w(x) = |2x-6| = |2(x-3)|$$



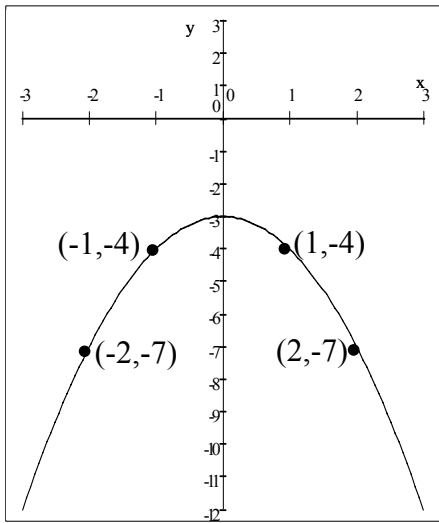
Factor out the 2

so $2x - 6$ becomes $2(x-3)$

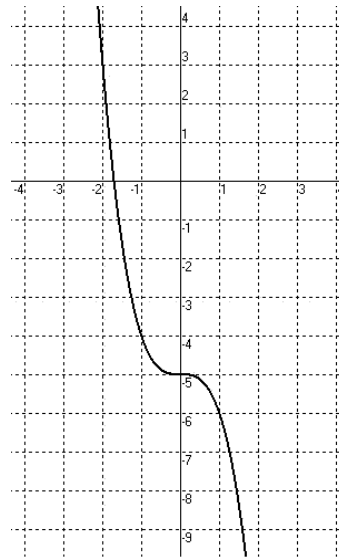
Shift right 3 units

Horizontal shrink by factor of 1/2
about the line $x = 3$

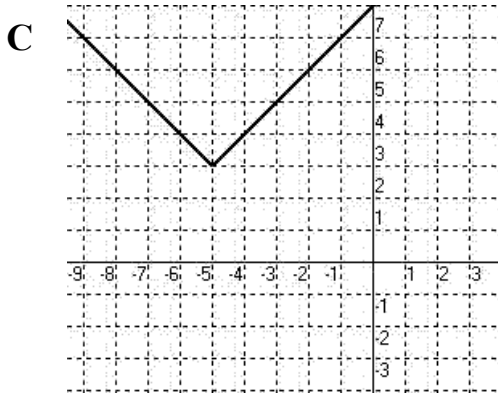
Transformations of Functions: Finding Equations From Graphs



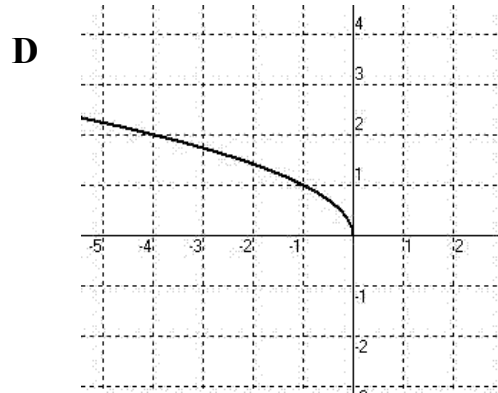
Equation $y = -x^2 - 3$



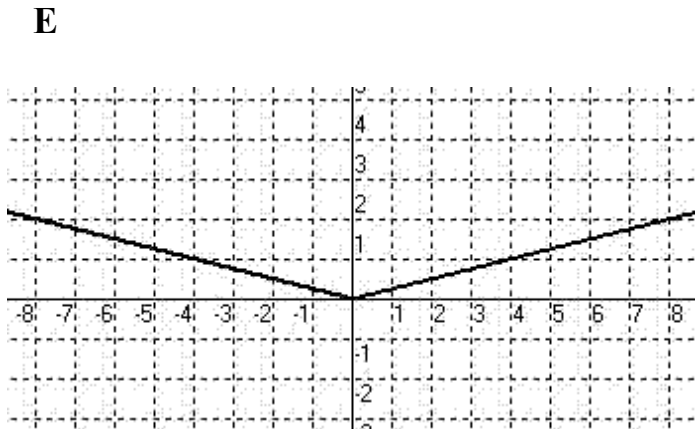
Equation $y = -(x)^3 - 5$ or $-x^3 - 5$



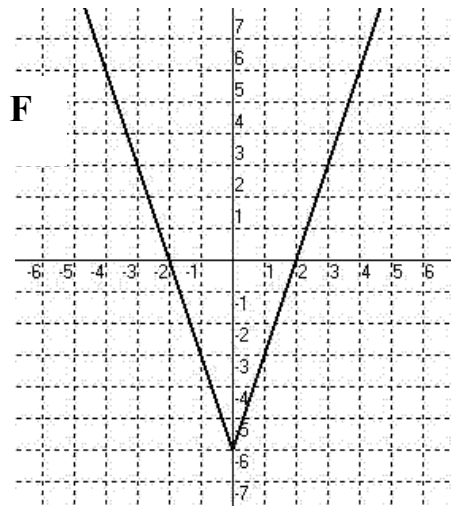
Equation $y = |x+5|+3$



Equation $y = \sqrt{-x}$



Equation $y = |.25x|$ or $.25|x|$



Equation $y = |3x| - 6$ or $3|x| - 6$