# Worksheet #9 - Speed

February 8, 2006

This worksheet is to build a foundation to the first concept in calculus that you will learn. We will come back to this example over and over again.

- If you drove 200 miles in 4 hours, what is your average speed? What was the formula that you used? Be precise in defining what the variables of the formula are.

  *OK, this is straight forward, but it is important. Your average speed would be 50 miles per hour and the formula would be the distance travelled divided by the length of time.*

  $$Velocity = \frac{\Delta D}{\Delta t}$$

- Are you claiming that you travelled this speed the entire time?

  *This is just an average. This is not the speed (or velocity) that you were travelling the whole time. Over a four hour period, you averaged 50 mph. Your speedometer was telling you the actual speed - sometimes faster than 50 mph, sometimes slower.*

- If you row 1000 meters in 4.5 minutes, what is your average speed?

  *Very similar computation as above. Divide 1000 meters by 4.5 minutes gives you the average speed of 222.22 meters per minute. Again, you aren't saying you went this speed the entire time. The time frame is too long to expect that you were going that speed the whole time.*

- If you are on top of a 800 foot building and drop a ball, the formula for measuring the height (in feet) of the ball at any given time $t$ (in seconds) is:

  $$H = 800 - 16t^2$$

- Fill in the chart. Be sure to use plenty of decimal places.

| t | H |
|---|---|
| 0 | 800 |
| 3 | 656 |
| 6 | 224 |
| 6.01 | 222.0784 |
| 6.001 | 223.80798 |

- What are some of the average speeds that the ball is travelling?

  *Similar computations. You should also realize that this computation is exactly like when we were finding the slope of a line. You just have to use the data points that are relevant to the question. The negative sign in the answers means the height is decreasing.*

$$\textit{Average speed in first three seconds } = \frac{800-656}{0-3} = -48$$

$$\textit{Average speed in first six seconds } = \frac{800-224}{0-6} = -96$$

$$\textit{Average speed between the third and sixth second} = \frac{656-224}{3-6} = -144$$

  *Each of these answers has units of feet per second. Just like before, these are just averages - we can't say anything with these numbers about the actual speed of the ball.*

- What is the average speed of the ball between $t = 6$ and $t = 6.01$? What is the average speed of the ball between $t = 6$ and $t = 6.001$?

$$\text{Average speed in first three seconds } = \frac{224-222.0784}{6-6.01} = -192.16$$

$$\text{Average speed in first three seconds } = \frac{224-223.80798}{6-6.001} = -192.02$$

  *Important - the formula to compute the average is the same. But since the time interval is so small (0.01 seconds and 0.001 seconds), we know that the average speed is very, very close to the actual speed. This is what we will call the instantaneous rate of change. And we can go crazy and used the times of $t = 6$ and $t = 6.0000001$, the instantaneous rate at $t = 6$ would be even closer to -192 feet per second. This was the idea behind the worksheet that we were using the limit, as we get closer to 6, the value of the average rate of change becomes the instantaneous rate of change.*

- Write a statement the describes how to compute (estimate) the actual speed of the ball after it has fallen for 7 seconds using the formula for the average speed.

  *The speed of the ball (instantaneous rate of change) of the ball at $t = 7$ is*

$$V = \lim_{h\to 0} \frac{H(7+h)-H(7)}{h} \approx \frac{H(7+0.001)-H(7)}{0.001} \approx -224$$

Here are some points that you need to remember for the topic quiz.

- How do you compute the average rate of change?
- How do you estimate the instantaneous rate of change?
- What is the difference between the average rate of change and the instantaneous rate of change?
- What are some of the different statements that can be made if given the average rate of change or the instantaneous rate of change?